**A Review of Units and Magnitudes**

**PHYS 4270 / 5390 4.0**

Purists (instructors?) would have everyone use SI units for all calculations, but this isn’t practical. Each discipline finds it more convenient to adopt units that fit its “scale”. For example, the SI unit for distance is the metre, but this is hardly useful for solar-system-scale problems. For the solar system, the Astronomical Unit (AU) is often used. For stellar astronomy, the parsec (pc) is used [the distance an object would have if its trigonometric parallax from Earth’s orbit were exactly 1 arcsecond], while extragalactic astronomers user the megaparsec (Mpc = 106 pc) or even gigaparsec (Gpc = 109 pc). [I really don’t like the parsec as a unit. It is an anthropocentric unit; if we lived on any other planet, the length of the parsec would be different. Not so for the light year!]

Chapter 1 on Radiation provides a good reference for relevant units. “Luminosity” (*L*) refers to the total power emitted by an object measured in Watts (1W = 1 Joule/second). The “flux” (*f*) of an object refers to the energy (density) received from an object per second per square metre. In the absence of an absorbing medium between an object and observer, then *f* = *L* / 4 π *d* 2 where *d* is the object-observer distance.

1. The Sun’s luminosity is 3.828 × 1026 W.
2. What is the flux from the Sun at 1 AU, i.e., at Earth’s distance from the Sun [W/m2] [Remember, units are never italicized in astronomy!] This is called the solar constant and is important in climate modelling. [1363 W/m2]
3. What is the flux from the Sun at a distance of 1 pc? [Look up the dimensions of a pc. By the way, there is never any need to memorize constants in this class.] [3.20 × 10-8 W/m2]

Astronomers use a variety of units for wavelength. High-energy astronomers prefer to use an energy, e.g., keV, rather than wavelength (since Planck’s Law gives an equivalence between the photon’s energy and its frequency or wavelength. Optical and near-infrared (NIR) astronomers use the Angstrom (Å) or 10-10 m, the nanometre (nm) or 10-9 m, or the micrometer or micron or 10-6 m (or μm). Millimetre and radio astronomers can use wavelengths (mm, cm, or m), but sometimes prefer using “frequency” (Hz, e.g., MHz or GHz).

Be that as it may, astronomers of all types are often called upon to incorporate data from other wavelength regimes – “multi-messenger astronomy” – to provide a comprehensive picture of a physical process. This requires transforming or interpreting one discipline’s units to your own.

While flux is a critical quantity in physics, the monochromatic energy density (the Spectral Energy Distribution, SED), conveys even more information. The monochromatic energy density is designated:

*f*λ = d*f*/d*λ* where [*f*λ] = W/m2/nm and *f*ν = d*f*/dν where [*f*ν] = W/m2/Hz

[Here, “nm” can be any distance quantity, whatever is most convenient, so “m” or “μm” would be possible.]

Many optical and NIR astronomers used/continue to use the units of erg/s/cm2/Å for *f*λ.

1. Convert 1 erg/s/cm2/Å into W/m2/nm. [10-2]

The relationship between *f*λ and *f*ν can be derived: *f*λ = d*f*/d*λ* = df/dν × dν/d*λ* = *f*ν (ν/*λ*) [show this]. Now let’s check the units using an example: if [*f*λ ] = W/m2/nm and [*f*ν] = W/m2/Hz, then [ν/*λ*] must be Hz/nm. In other words, the *λ* in (ν/*λ*) has to be expressed in nm (in this case). So be careful.

1. Convert *f*λ = 3.63 × 10-11 W/m2/nm into *f*ν (W/m2/Hz) for a wavelength of 545 nm. [This turns out to be the monochromatic flux of the star Vega, an important flux calibration object in the 1960s and 1970s.] [3.59 × 10-23]
2. The Jansky (named after Karl Jansky), is an important long-wavelength flux density unit: 1 Jy = 10-26 W/m2/Hz. Convert *f*ν = **100 Jy** into *f*λ (W/m2/μm) at 2.2 μm. [6.20 × 10-11]

There are some types of detector that can measure the energy density directly, but CCDs and NIR detectors actually count photons. Astronomers need to know how to convert energy flux ↔ photon flux. This is straightforward incorporating Planck’s Law. So *p*ν= (*f*ν / hν) and *p*λ= (*f*λ/ hc/*λ*). Clearly, [*p*λ ] = photons/s/m2/nm, etc.

1. Convert *f*λ and *f*ν from question 4 into photon fluxes, *p*λ and *p*ν. [about 106 photons/s/m2/nm, 10-5 photons/s/m2/Hz]

A “magnitude” is a measurement of brightness in optical/NIR astronomy. The modern magnitude system was based in part on the original system of Hipparchus (2nd century BC), and with the observation in the 19th century that the eye is a quasi-logarithmic detector (as is the ear). (This is fortunate since the human eye can detect the faintest LED in the dark at one moment and mid-day ski slopes at another.) The definition of the (apparent) magnitude, m = -2.5log10(*f*) + *Q*, where m is the apparent magnitude, *f* is the energy or photon flux and *Q* is the zero point that depends on the photometric system. Note that *f*, m can be a monochromatic flux & magnitude, or an flux density & magnitude (in a finite or even infinite waveband). The most useful format for the apparent magnitude equation is,

m1 – m2 = -2.5 log10 (*f*1/*f*2)

where “1” and “2” can refer to two separate objects, or to two wavebands (part of the SED) in the same object.

1. What is the ratio of the brightnesses (to five significant figures) of two objects that differ in apparent magnitude by exactly one magnitude? [2.5119]
2. What is the apparent magnitude difference between two objects that have a brightness ratio of 0.1234? (Magnitudes are dimensionless – the units are taken up in the zero point. Magnitudes are reported to two, or at most, three decimal places. Never more. As the notes show, an uncertainty of 0.01 magnitudes is an uncertainty of 1% in linear units.) [+2.27]

The other magnitude equation involves the distance to an object relative to a fiducial distance of 10 pc. The apparent magnitude an object has if it were at 10 pc is called the absolute magnitude, M. The distance modulus: m – M = 5 log10 (*d*/10 pc), where *d* is the distance in pc. Notice that the distance modulus involves only magnitudes on the left-hand side, and distance on the right.

1. What is the distance modulus of a star whose distance is 32 pc? [+2.53]
2. If the apparent magnitude of the object is +6.71, what is its absolute magnitude? [+4.18]
3. If the object was twice as far away, what is the difference in the distance modulus compared to the result in (a)? [+1.50]
4. The Sun has an apparent magnitude of -26.74 (don’t worry about filters and wavebands just yet). It also has an absolute magnitude of +4.83.
5. What is the distance modulus of the Sun? [-31.57]
6. What would be the Sun’s apparent magnitude at a distance of α Cen (whose distance is 4.367 light years)? [+0.46]
7. What would be the apparent magnitude of the Sun at a distance of 8 kpc (8,000 pc) assuming no attenuation of the light (due to intervening dust and gas)? [+19.35]
8. The “AB magnitude” system is often used in contemporary astronomy (e.g., the Sloan Digital Sky Survey uses a magnitude system that is “almost the AB system”). While the expression in our notes reflects the correct expression for the AB monochromatic magnitude, it is actually defined as:

mAB = -2.5 log10 (*f*ν) – 48.60 where [*f*ν] = erg/s/cm2/Hz and 48.60 is the photometric constant

1. If *f*ν were measured in W/m2/Hz, what would the photometric constant become? [-56.10]